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# DISCRETE-TIME PHYSICS-INFORMED NEURAL NETWORKS MODELLING FOR 1D CONSOLIDATION

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## 1 Abstract

Numerical modelling of coupled hydro-mechanical (HM) processes in porous materials presents a significant challenge, primarily due to their intricate physical nature, resulting in governing equations with complex initial and boundary conditions (IC/BCs) expressed as nonlinear partial differential equations (PDEs). Physics-informed neural networks (PINNs) have emerged as an innovative approach to solving PDEs while effectively handling the IC/BCs within a single computational algorithm, applicable to both forward and inverse problems. The framework is used to develop surrogate models of problems, making it suitable for scenarios where direct numerical simulations are computationally demanding. The present paper aims to investigate the application of the PINNs in addressing fundamental coupled HM one-dimensional consolidation problems in porous materials. Compared to analytical solutions, simulations results revealed that the PINN model can accurately capture the hydro-mechanical behaviour of the porous materials.

## 2 Research method

In the present study, a discrete-time physics-informed neural network (PINN) is employed to model the one-dimensional consolidation problem, as shown in Figure 1. In this approach, time, as an independent variable, is discretized into consecutive intervals in which the unknowns of the problem are approximated by neural networks (NNs). As time is assumed to be constant within each interval, the input to the NNs consists solely of spatial coordinates, resulting in one order reduction of the input-space dimension. This highlights the main difference between discrete- and continuous-time PINNs, where time is also included as an input to the NNs.

Moreover, the parameters of these NNs are determined through an iterative optimization algorithm, known as training, in each time interval. In conventional machine learning and deep learning methods, the loss function is typically based on the sum of squared errors, requiring empirical data. In contrast, the PINN framework incorporates the known physics of the problem, including governing equations (GEs) and initial and boundary conditions (IC/BCs), into the loss function as additional regularization terms (Nguyen et al., 2022; Raissi et al., 2019; Raissi & Karniadakis, 2018). Although the PINN framework can incorporate empirical data, this study aims to solve the 1D consolidation problem using a data-free approach, relying solely on physical constraints.

### *2.1 Governing equations, boundary and initial conditions*

The governing equations (GEs) associated with the one-dimensional consolidation problem are conservation of linear momentum in x-direction (see Figure 1) and pore fluid mass as stated in Eqs. (1) and (2).

$$\frac{3(1-\upsilon)}{1+\upsilon} K_{dr} \frac{\partial^2 u}{\partial x^2} - \alpha \frac{\partial p}{\partial x} = 0 \tag{1}$$

$$\frac{1}{Q}\frac{\partial p}{\partial t} - \frac{k}{\mu}\frac{\partial^2 p}{\partial x^2} + \alpha \frac{\partial^2 u}{\partial t\, \partial x} = 0 \tag{2}$$

where $u$ and $p$ are displacement and pore fluid pressure, $K_{dr}$ and $\upsilon$ are bulk modulus and Poisson's ratio of the porous medium in drained condition, and $k$ and $\mu$ are intrinsic permeability and fluid viscosity, respectively. Moreover, $\alpha$ and $Q$ are Biot's coefficients determined according to fluid and solid grains bulk moduli (Zienkiewicz et al., 1980).
In addition to the proposed GEs stated in Eqs. (1) – (2), the IC/BCs of the problem for time interval, $t_i$, are as follows

$$\sigma_{xx}(0) = \sigma_0 \tag{4}$$

$$u(L_x) = 0 \tag{5}$$

$$p(0) = 0 \tag{7}$$

$$\partial p / \partial x\, (L_x) = 0 \tag{8}$$

in which $\sigma_{xx}$ and $\sigma_0$ are the total stress in x-direction and the applied load as shown in Figure 1.

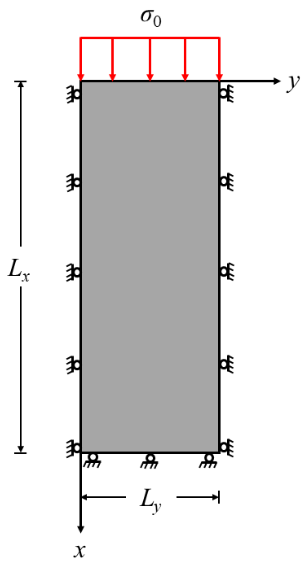

Figure 1- One-dimensional consolidation

### 2.2 Discrete-time physics-informed neural networks framework for consolidation

To apply the discrete-time PINN to the 1D consolidation problem, the total time of the problem is divided into multiple time intervals ($t_0, t_1, t_2, etc.$). Let us consider an arbitrary time interval beginning at $t_i$ and ends at $t_i + \Delta t$. It is assumed that the quantities ($\hat{p}, \hat{u}$) and their partial derivatives with respect to ($\bar{x}, \bar{t}$) are known, meaning that the ICs of the current time interval are determined. For this time interval, the primary unknowns including pore fluid pressure and displacements, are approximated using fully connected multilayer perceptrons (MLPs) with the tanh activation function. Since the unknowns are functions of the spatial coordinate, $\bar{x}$, the first step is to generate the input training set by randomly selecting points from the spatial domain (The $\bar{\square}$ denotes normalized variables to avoid numerical issues during computation). The loss function associated with present problem can be evaluated according to Eqs. (1) – (8) as specified in Eq. (9).

$$Loss(t_i) = \sum_j \lambda_j\, Loss_j \tag{9}$$

where $Loss_j$ indicates individual loss terms, and $\lambda_j$s are adaptive weights determined according to GradNorm algorithm (Chen et al., 2018).

It is worth noting that since the initial conditions (ICs) for the current time interval are assumed to be the final state of $(\hat{\bar{p}}, \hat{\bar{u}})$ from the previous time interval, the corresponding loss term is determined accordingly.
Following that, by approximating $(\hat{\bar{p}}, \hat{\bar{u}})$, the loss value can be evaluated using Eq. (9). Notably, the partial derivatives of $(\hat{\bar{p}}, \hat{\bar{u}})$ with respect to independent variables, $(\bar{x}, \bar{t})$, can be obtained using automatic differentiation technique implicitly implemented in open-source python packages (Haghighat & Juanes, 2021; Paszke et al., 2017).

## 3 Results

Figure 2 shows the training history and prediction results of the PINN model along with the analytical solution (Terzaghi, 1943). According to Figure 2, it can be concluded that the PINN model is able to capture the true solution with the average relative errors of 1.64% and 0.26% for normalized pore fluid pressure and displacement. It should be highlighted that the displacement values are normalized with respect to ultimate deformation, $u_f$.
Moreover, to further examine the PINN accuracy, the model predictions are evaluated several times including $\bar{t} = 0.0001, 0.005, 0.05, 0.2, 1.0$ as depicted in Figure 3 and it can be observed that the PINN predictions are properly matched with the exact solution.

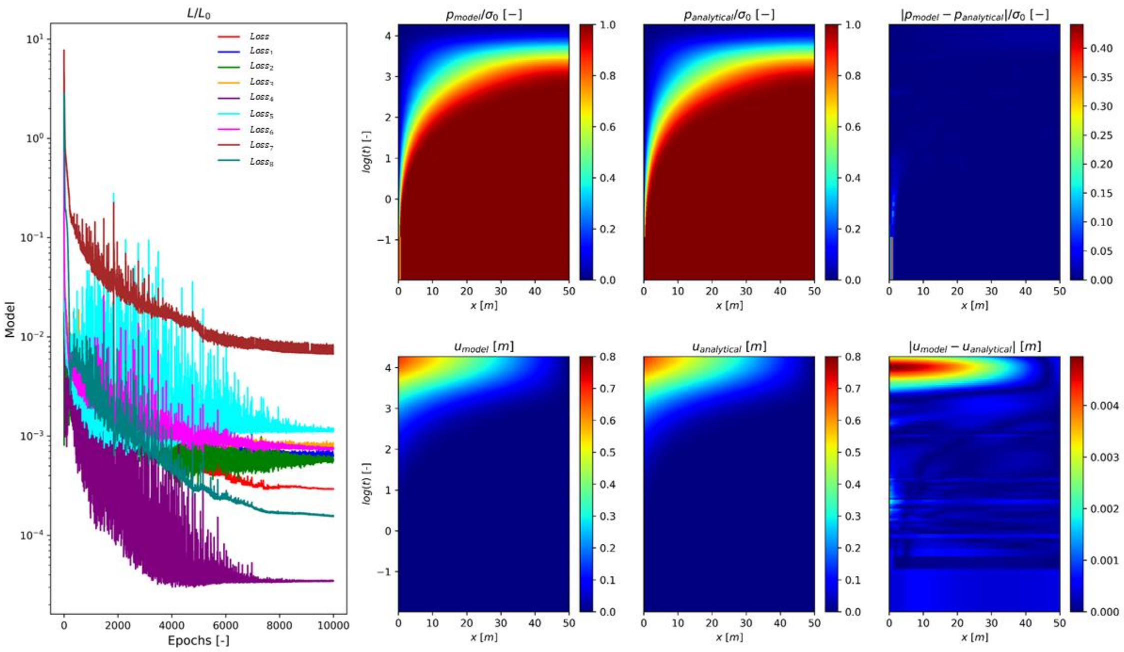


Figure 2- Training and prediction results of PINN model vs Analytical solution for one-dimensional consolidation

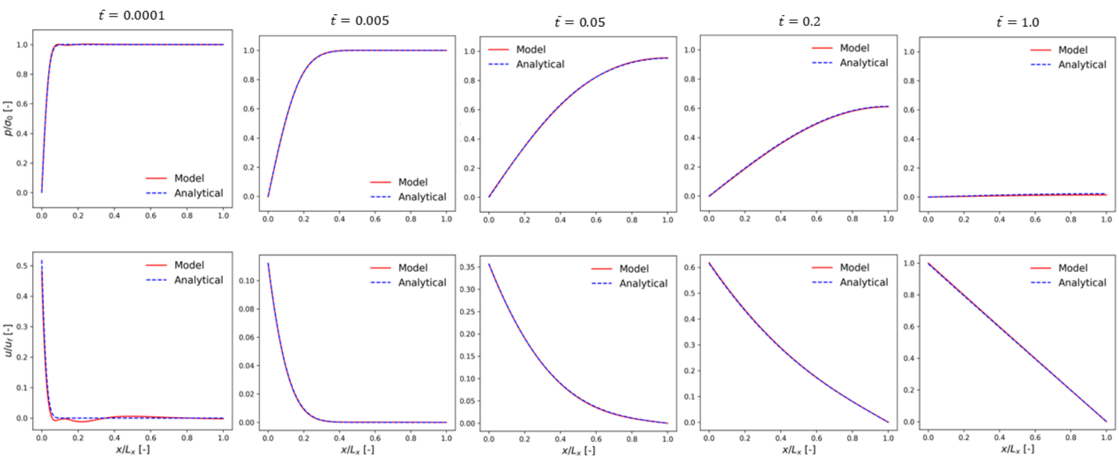


Figure 3- PINN model predictions for several times for one-dimensional consolidation

## 4 Conclusion

Although physics-informed neural networks (PINNs) have been the subject of research in various fields of science and engineering, modelling the hydro-mechanical processes in porous materials remains challenging. In this study, the one-dimensional consolidation problem was modelled using the discrete-time PINN framework, achieving results that closely match the analytical solution, with average relative errors of 1.64% and 0.26% for normalized pore fluid pressure and displacements, respectively.

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